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Decision Support

The distribution of power in the European Constitution

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Abstract

The aim of this paper is to analyze the distribution of voting power in the Constitution for the enlarged European Union. By using generating functions, we calculate the Banzhaf power indices for the European countries in the Council of Ministers under the decision rules prescribed by the Treaty of Nice and the new rules proposed by the European Constitution Treaty. Moreover, we analyze the power of the European citizens under the egalitarian model proposed by Felsenthal and Machover [D.S. Felsenthal, M. Machover, The measurement of voting power: Theory and practice, problems and paradoxes, Edward Elgar, Cheltenham, 1998].

Keywords: Power indices; Banzhaf index; European Constitution

1. Introduction

The weighted voting games are mathematical models which are used to analyze the distribution of the decision power of a nation in a supranational organization like the Council of Ministers of the European Union, the Security Council of the United Nations or the International Monetary Fund. In these institutions, each nation has associated a number of votes and a proposal is approved if a coalition of nations has enough votes to reach an established quota. For example, the voting method of the Security Council of the United Nations, formed by 5 permanent members and 10 temporary, is the game in which each one of the permanent nations (the United States, Russia, China, the United Kingdom and France) has seven votes and each one of the temporary an only one vote, being the established quota 39 votes. Let us observe that any coalition that does not include some of the five permanent nations has at most $(4 \times 7) + 10$ votes, which is an inferior number to the fixed quota; so this coalition will not be a winning coalition. Therefore, the permanent members have capacity to veto any proposal.

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The power of a country in a supranational organization is a numerical measure of its capacity to decide the approval of a motion. This decisive character is measured calculating the number of times that the vote of a country converts to a coalition that does not reach the quota to take decisions in a winning coalition. The power indices are a priori measures of this power, the most useful are the Shapley–Shubik [16] and Banzhaf [2] indices. Both of them provide a much exacter measure of the power of a player than the number of votes that this player is entitled to cast. Another planned question in the decision-making is the following: How is the power of a country measured to block a decision? The answer to this question is that the power of a country to block decisions is the same as it has to approve them. In other words, both the Banzhaf index and Shapley–Shubik index coincide in block and approval situations. Thus, these indices measure both the capacity of a country to approve a proposal and block it. In this paper, we analyze the voting power of a country in the Council of the European Union by using the Banzhaf index. We justify our preference for the Banzhaf index in the model of *I-power* (Power as Influence) defined by Felsenthal and Machover [8, Chapter 3], and also in the empirical results obtained by Leech [14].

One of the fundamental agreements of the Intergovernmental Conference of the European Union, which took place in Nice in December 2000, was the approval of new voting systems in order to improve the decision rules for the enlargement of the European Union. Several voting systems were discussed to take decisions in the Council of the European Union, where two models of triple majority with a new weighting of the votes were approved. These models correspond to weighted votes, number of countries and population. The Nice rules were established to enlarge the European Union to 25 countries, so the total number of coalitions is larger than 33 million. For that, the line of reasoning only based on the analysis of a fewness winning coalitions is not a rational method. For example, for the first rule approved in Nice, Germany is decisive in more than 900,000 coalitions.

The new voting rule proposed by the European Convention for the future European Constitution changes in a very remarkable way the power of the countries in the Council. The reason is that the weighted votes, that were approved in Nice are removed and a coalition only needs 15 votes, which at least sum up by 65% of the population to approve a decision with the new rule. Furthermore, the minimum number of countries to block a proposal is four and the abstentions are not counted.

Let us outline the contents. Section 2 briefly recalls the concepts of weighted multiple majority games and introduces the voting game for the European Council of Ministers proposed in the Constitution for Europe. In Section 3, we obtain the algorithms to compute the swings and the Banzhaf power index by using generating functions. In Section 4, we apply these algorithms to compute the Banzhaf index under the Nice, the European Convention and the European Constitution rules, which will be used in the European Union enlarged to 25 countries. Finally, Section 5 is devoted to analyzing the model of the square root of the population proposed by Felsenthal and Machover [8].

2. A model of game for the European Constitution

Definition 1. A simple game is a pair (N, v) where $N = \{1, \ldots, n\}$ is the set of players and v: $2^N \to \{0, 1\}$ is the characteristic function which satisfies $v(\emptyset) = 0$, $v(N) = 1$ and $v(S) \leqslant v(T)$ whenever $S \subseteq T$. A coalition of players $S \subseteq N$ is winning if $v(S) = 1$, and coalitions with $v(S) = 0$ are called losing. A coalition B is blocking if $N \backslash B$ is losing.

A simple game (N, v) is *proper* if $v(S) + v(N \backslash S) \leqslant 1$ for all $S \subseteq N$. If equality holds for every coalition S, the game is said to be *decisive*. In an improper game there will be at least one pair of non-intersecting winning coalitions.

We introduce a special class of simple games called *weighted voting games*. The symbol $[q; w_1, \ldots, w_n]$ will be used, where q and $w_1, \ldots, w_n$ are positive integers with

$$w_i < q \leqslant \sum_{i=1}^{n} w_i \quad \text{for } i = 1, \dots, n.$$

Here, there are n players, w_i is the number of votes of player i, and q is the needed quota so that a coalition can win. The symbol $[q;\ w_1, \dots, w_n]$ represents the simple game (N, v) defined by:

$$v(S) = \begin{cases} 1 & \text{if } w(S) \geqslant q, \\ 0 & \text{if } w(S) < q, \end{cases}$$

where $S \subseteq N$ and $w(S) = \sum_{i \in S} w_i$. If the quota $q > w(N)/2$ then the weighted voting game is proper. Otherwise, S and $N \backslash S$ would be winnings, i.e., $w(S) \geqslant q$ and $w(N \backslash S) \geqslant q$. Thus,

$$w(N) = w(S) + w(N \setminus S) \geqslant 2q > w(N),$$

which is a contradiction.

Example 1. The game $[5;\ 4, 3, 2, 1]$ is improper and the game $[6;\ 4, 3, 2, 1]$ is proper but it is not decisive.

Given the simple games $(N, v_1), \dots, (N, v_m)$ we now consider the simple games $(N, v_1 \wedge \cdots \wedge v_m)$ and $(N, v_1 \vee \cdots \vee v_m)$ defined by:

$$(v_1 \wedge \cdots \wedge v_m)(S) = \min\{v_t(S) : 1 \leqslant t \leqslant m\},$$
$$(v_1 \vee \cdots \vee v_m)(S) = \max\{v_t(S) : 1 \leqslant t \leqslant m\}.$$

A *weighted m-majority game* is the simple game $(N, v_1 \wedge \cdots \wedge v_m)$ where the games (N, v_t) are the weighted voting games represented by

$$[q^t;\ w_1^t, \dots, w_n^t]$$

for $1 \leqslant t \leqslant m$. Then, its characteristic function is given by:

$$(v_1 \wedge \cdots \wedge v_m)(S) = \begin{cases} 1 & \text{if } w^t(S) \geqslant q^t,\ 1 \leqslant t \leqslant m, \\ 0 & \text{otherwise,} \end{cases}$$

where $w^t(S) = \sum_{i \in S} w_i^t$.

Remark 1. If $m = 2$ or $m = 3$ then we obtain weighted double or triple majority games, respectively.

The Nice European Council in December 2000 established the decision rule for the EU enlarged to 25 countries. This rule is contained in the *Declaration on the enlargement of the European Union* and the *Declaration on the qualified majority threshold and the number of votes for a blocking minority in an enlarged Union* (Official Journal of the European Communities 10.3.2001, C 80/80-85).

The players in the Council of the EU enlarged to 25 countries and the corresponding population weights are showed in Table 1. The data of population used to calculate the mentioned weights are those provided by the Office of the Census of Eurostat corresponding to January 2003.

In the next proposition we obtain the simple game associated to the Nice rule.

Proposition 1. *The Nice rule is the weighted triple majority game* $v_1 \wedge v_2 \wedge v_3$, *where*

$$v_1 = [232;\ 29, 29, 29, 29, 27, 27, 13, 12, 12, 12, 12, 12, 10, 10, 7, 7, 7, 7, 7, 4, 4, 4, 4, 4, 3],$$
$$v_2 = [13;\ 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1],$$
$$v_3 = [620;\ 182, 131, 130, 126, 91, 84, 36, 24, 23, 23, 22, 22, 20, 18, 12, 12, 11, 9, 8, 5, 4, 3, 2, 1, 1].$$

Table 1
Population in January 2003 and population weights for the 25 EU members

Countries	Population	Weights
Germany	82,536,700	182
France	59,630,100	131
United Kingdom	59,328,900	130
Italy	57,321,000	126
Spain	41,550,600	91
Poland	38,218,500	84
The Netherlands	16,192,600	36
Greece	11,018,400	24
Portugal	10,407,500	23
Belgium	10,355,800	23
Czech Republic	10,203,300	22
Hungary	10,142,400	22
Sweden	8,940,800	20
Austria	8,067,300	18
Denmark	5,383,500	12
Slovak Republic	5,379,200	12
Finland	5,206,300	11
Ireland	3,963,600	9
Lithuania	3,462,600	8
Latvia	2,331,500	5
Slovenia	1,995,000	4
Estonia	1,356,000	3
Cyprus	715,100	2
Luxembourg	448,300	1
Malta	397,300	1

Notice that the game v_3 is defined assigning to every country, a number of votes equal to the rate per thousand of its population over the total population of the European Union and the quota represents 62% of the total population. So, a voting will be favourable if it counts on the support of 13 countries with at least 232 votes, and with at least 62% of the population.

The voting method approved in the summit of Brussels on 18th June, 2004, for its incorporation to the European Constitution, is more complex because it is based on a double voting system and a blocking clause. To approve a proposal in the Council of Ministers of the 25 members of the European Union, it is needed at least 15 countries that sum up more or equal than 65% of the population. Moreover, the minimum number of countries to block a proposal is four and the abstentions are not counted.

Proposition 2. *The European Constitution rule is the game* $(v_2' \wedge v_3') \vee bc$, *where*

$$v_2' = [15;\ 1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1],$$
$$v_3' = [650;\ 182,131,130,126,91,84,36,24,23,23,22,22,20,18,12,12,11,9,8,5,4,3,2,1,1],$$
$$bc = [22;\ 1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1].$$

Proof. In the weighted double majority game $v_2' \wedge v_3'$, a coalition S is winning if $|S| \geqslant 15$ and $w(S) \geqslant 650$, where $w_1, \ldots, w_{25}$ are the population weights. Then B is a blocking coalition if $N \backslash B$ is losing, i.e., $|N \backslash B| < 15$ or $w(N \backslash B) < 650$. This is equivalent to $|B| > |N| - 15 = 10$ or $w(B) > w(N) - 650 = 350$. Furthermore, in the European Constitution (EC) game the minimum number of countries to block a proposal is four. Thus, a coalition B is blocking in the EC game if

$$(|B| > 10 \quad \text{or} \quad w(B) > 350) \quad \text{and} \quad |B| > 3.$$

Then a coalition S is winning in the EC game if $N\backslash S$ is not EC-blocking, that is,

$$(|N \setminus S| \leqslant 10 \quad \text{and} \quad w(N \setminus S) \leqslant 350) \quad \text{or} \quad |N \setminus S| \leqslant 3,$$

which is equivalent to

$$(|S| \geqslant 15 \quad \text{and} \quad w(S) \geqslant 650) \quad \text{or} \quad |S| \geqslant 22$$

and hence we obtain the game $(v_2' \wedge v_3') \vee bc$. □

3. Generating functions to compute swings

The Banzhaf power index is concerned with the number of times each player could change a coalition from losing to winning and it requires to know the number of swings for every player i (see Dubey and Shapley [6]). A *swing* for player i is a pair of coalitions $(S \cup \{i\}, S)$ such that $S \cup \{i\}$ is winning and S is not. For each $i \in N$, we denote by $b_i(v)$ the number of swings for i in the game v, i.e., the number of winning coalitions in which player i is critical. The total number of swings is

$$\bar{b}(v) = \sum_{i \in N} b_i(v).$$

Definition 2. The normalized Banzhaf index is the vector $\beta(v) \in \mathbb{R}^n$ where

$$\beta_i(v) = \frac{b_i(v)}{\bar{b}(v)}, \quad 1 \leqslant i \leqslant n.$$

This Banzhaf power index depends on the number of ways in which each voter can effect a swing. If there are n players in a voting situation, then the function which measures the worst case running time for computing these indices is in $O(n2^n)$.

We use a combinatorial method based on *generating functions* given by Bilbao et al. [1,3] to calculate the normalized Banzhaf index in pseudo-polynomial time. With this method we obtain the Banzhaf power indices efficiently in the weighted triple and double majority games prescribed by the Treaty of Nice and proposed by the European Constitution, respectively. We first present two results on generating functions for computing the Banzhaf power index in weighted double majority games (see Bilbao et al. [3]).

The most useful method for counting the number of elements $f(k)$ of a finite set is to obtain its generating function. The *generating function* of $f(k)$ is the formal power series

$$\sum_{k \geqslant 0} f(k) x^k.$$

We will use generating functions of several variables like

$$\sum_{k \geqslant 0} \sum_{j \geqslant 0} f(k,j) x^k y^j.$$

Brams and Affuso [4] obtained generating functions for computing the normalized Banzhaf index. Let $v = [q; w_1, \ldots, w_n]$ be a weighted voting game. They noted that the number of swings for player i satisfies

$$\eta_i(v) = |\{S \subseteq N : v(S) = 0,\ v(S \cup \{i\}) = 1\}| = \sum_{k=q-w_i}^{q-1} b_k^i,$$

where b_k^i is the number of coalitions S such that $i \notin S$ with $w(S) = k$.

Proposition 3 (Brams–Affuso). *Let $v = [q;\ w_1, \ldots, w_n]$ be a weighted voting game. Then the generating functions of numbers $\{b_k^i\}$ are given by*

$$B_i(x) = \prod_{j=1, j\neq i}^{n} (1 + x^{w_j}).$$

We now present generating functions for computing the Banzhaf power index for weighted double majority games.

Proposition 4 (Bilbao et al.). *Let (N, v) be a weighted double majority game with $v = v_1 \wedge v_2$, where $v_1 = [q; w_1, \ldots, w_n]$ and $v_2 = [p; p_1, \ldots, p_n]$. For every $i \in N$, the number of swings for player i is given by*

$$\eta_i(v) = \sum_{k=q-w_i}^{w(N\setminus i)} \sum_{r=p-p_i}^{p(N\setminus i)} b_{kr}^i - \sum_{k=q}^{w(N\setminus i)} \sum_{r=p}^{p(N\setminus i)} b_{kr}^i,$$

where b_{kr}^i is the number of coalitions S such that $i \notin S$ with $w(S) = k$ and $p(S) = r$.

We also establish a generating function to obtain the numbers $\{b_{kr}^i\}_{k,r\geqslant 0}$.

Proposition 5 (Bilbao et al.). *Let (N, v) be a weighted double majority game with $v = v_1 \wedge v_2$, where $v_1 = [q; w_1, \ldots, w_n]$ and $v_2 = [p; p_1, \ldots, p_n]$. Then, for each $i \in N$, the generating function of $\{b_{kr}^i\}_{k,r\geqslant 0}$, where b_{kr}^i is the number of coalitions $S \subseteq N$ such that $i \notin S$, $w(S) = k$ and $p(S) = r$ is given by*

$$B_i(x, y) = \prod_{j=1, j\neq i}^{n} (1 + x^{w_j} y^{p_j}).$$

By using the above generating function we compute the number of swings for player i in the European Constitution game $(v_2' \wedge v_3') \vee bc$.

Theorem 6. *Let (N, v) be weighted double majority game with blocking given by $v = (v_2' \wedge v_3') \vee bc$, where $v_2' = [p; 1, \ldots, 1]$, $v_3' = [q; w_1, \ldots, w_n]$, $bc = [|N| - b; 1, \ldots, 1]$ and $p < |N| - b$. For every $i \in N$, the number of swings for player i is given by*

$$\eta_i(v) = \sum_{k=q-w_i}^{q-1} \sum_{r=p}^{|N|-b-2} b_{kr}^i + \sum_{k=q-w_i}^{w(N\setminus i)} b_{k,p-1}^i + \sum_{k=1}^{q-1} b_{k,|N|-b-1}^i,$$

where b_{kr}^i is the number of coalitions S such that $i \notin S$ with $w(S) = k$ and $|S| = r$.

Proof. We consider the set of all coalitions S such that $i \notin S$, $v(S \cup \{i\}) = 1$, and $v(S) = 0$. For the game $v = (v_2' \wedge v_3') \vee bc$, we obtain

$$(w(S \cup \{i\}) \geqslant q \quad \text{and} \quad |S \cup \{i\}| \geqslant p) \quad \text{or} \quad |S \cup \{i\}| \geqslant |N| - b$$

and also

$$(w(S) < q \quad \text{and} \quad |S| < |N| - b) \quad \text{or} \quad (|S| < p \quad \text{and} \quad |S| < |N| - b).$$

Since $p < |N| - b$, the above conditions are equivalent to

$$(w(S) \geqslant q - w_i \quad \text{and} \quad |S| \geqslant p - 1) \quad \text{or} \quad |S| \geqslant |N| - b - 1$$

and also

$$(w(S) \leqslant q - 1 \quad \text{and} \quad |S| \leqslant |N| - b - 1) \quad \text{or} \quad |S| \leqslant p - 1.$$

These conditions are equivalent to (1) ∨ (2) ∨ (3), where

$$q - w_i \leqslant w(S) \leqslant q - 1 \quad \text{and} \quad p - 1 \leqslant |S| \leqslant |N| - b - 1, \tag{1}$$

$$q - w_i \leqslant w(S) \quad \text{and} \quad |S| = p - 1, \tag{2}$$

$$w(S) \leqslant q - 1 \quad \text{and} \quad |S| = |N| - b - 1. \tag{3}$$

Note that condition $|N| - b - 1 \leqslant |S| \leqslant p - 1 < |N| - b - 1$ is a contradiction. The number of coalitions S such that (2) is satisfied and $i \notin S$, is given by

$$s_2^i = \sum_{k=q-w_i}^{w(N \setminus i)} b_{k,p-1}^i.$$

The cardinality of the set of coalitions S such that (3) is satisfied and $i \notin S$, is

$$s_3^i = \sum_{k=1}^{q-1} b_{k,|N|-b-1}^i.$$

Since the coalitions S such that $|S| = p - 1$ or $|S| = |N| - b - 1$ in condition (1) are considered in (2) and (3), respectively, we conclude that the number of coalitions S such that (1) is satisfied, (2) and (3) are not true, and $i \notin S$, is

$$s_1^i = \sum_{k=q-w_i}^{q-1} \sum_{r=p}^{|N|-b-2} b_{kr}^i.$$

Therefore, the number of swings for player i is $\eta_i(v) = s_1^i + s_2^i + s_3^i$. □

The European Constitution game is the game defined in Theorem 6 where $|N| = 25$, $p = 15$, $q = 650$, and $b = 3$. The algorithm showed in Theorem 6, written in the Mathematica program [18], to compute the swings in the European Constitution game is the following:

```
banzhafTwoG[weights_List,mem_List]:=Times @@ (1+x^weights y^mem)
banzhafTwoBlocking[i_,weights_List,mem_List,q_,p_,b_]:=
Module[{g,coefi,n,m,s1,s2,s3},
g=banzhafTwoG[Delete[weights,i],Delete[mem,i]];
n=Exponent[g,x]+1; m=Exponent[g,y]+1;
coefi=CoefficientList[g,{x,y}]/.{}→Table[0,{m}];
s1=Apply[Plus,Flatten[coefi[[Range[q-weights[[i]]+1,n],Range[p,p]]]]];
s2=Apply[Plus,Flatten[coefi[[Range[q-weights[[i]]+1,q],Range[p+1,Apply
[Plus,mem]-b-1]]]]];
s3=Apply[Plus,Flatten[coefi[[Range[1,q],Range[Apply[Plus,mem]-b,Apply
[Plus,mem]-b]]]]];
s1+s2+s3]
SwingsTwoBlocking[weights_List,mem_List,q_,p_,b_]:=
Table[banzhafTwoBlocking[i,weights,mem,q,p,b],{i,Length[weights]}]
SwingsTwoBlocking[weights_List,mem_List,650,15,3]
```

4. The power of the European countries

The Council of Ministers of the EU represents the national governments of the member states. The Council uses a voting system of qualified majority to pass new legislation. Felsenthal and Machover [9] analyzed in terms of a priori measures of power these decision rules for the Council of Ministers of the EU. They used the Bräuninger–König IOP 1.0 program and the Lemma 3.3.12 in Felsenthal and Machover [8] to calculate the voting power of each one of the present 15 members and the future 25 ones. Moreover, Felsenthal and Machover use the new version of the program IOP 2.0 (see Bräuninger and König [5]) to calculate voting power indices for the post Nice institutions in the 25-member and 27-member scenarios (see [10,11]). The order of time complexity of the method described in Section 3 is n^2C, where C is the number of non-zero coefficients of the generating function and improves the computation time (see Bilbao et al. [3]). Furthermore, the generating function method allows us to calculate the exact number of swings in two scenarios: the double majority game without the blocking clause $v_2' \wedge v_3'$ and the European Constitution game $(v_2' \wedge v_3') \vee bc$. The obtained data prove that this clause has an insignificant impact in the distribution of power in the Council of Ministers.

The swings of the countries in the double majority game $v_2' \wedge v_3'$ and in the European Constitution game $(v_2' \wedge v_3') \vee bc$, are showed in the columns second and fourth of Table 2. In the third column, the differences of swings are indicated. These data show that the requiring clause "at least four countries to block" has an irrelevant impact in the distribution of the power. Germany loses eight swings, France and the United Kingdom keep the same swings, Italy and Spain win two and Poland four. These profits or losses of the countries with more population are summed up or subtracted to more than a million of swings. Moreover, the 19 countries with smaller population win 12 swings that add to more than half million of swings that these countries have in the double voting game without the blocking clause.

In Table 3, the swings of the 25 nations of the European Union and the two candidate nations (Romania and Bulgaria) are showed. In this scenario, Germany also loses eight swings, France, the United Kingdom and Italy still have the same ones, Spain and Poland win four; while the smallest 21 countries win 10 swings.

Table 2
Swings for the 25 EU members

Countries	Double game	Difference	EC game
Germany	2,668,027	−8	2,668,019
France	1,940,159	0	1,940,159
United Kingdom	1,929,669	0	1,929,669
Italy	1,889,319	+2	1,889,321
Spain	1,490,413	+2	1,490,415
Poland	1,422,169	+4	1,422,173
The Netherlands	962,829	+12	962,841
Greece	851,897	+12	851,909
Portugal	842,213	+12	842,225
Belgium	842,213	+12	842,225
Czech Republic	832,527	+12	832,539
Hungary	832,527	+12	832,539
Sweden	813,199	+12	813,211
Austria	793,799	+12	793,811
Denmark	736,231	+12	736,243
Slovak Republic	736,231	+12	736,243
Finland	726,573	+12	726,585
Ireland	707,217	+12	707,229
Lithuania	697,547	+12	697,559
Latvia	668,999	+12	669,011
Slovenia	659,337	+12	659,349
Estonia	649,681	+12	649,693
Cyprus	640,093	+12	640,105
Luxembourg	630,555	+12	630,567
Malta	630,555	+12	630,567

Table 3
Swings for the 27 EU members

Countries	Double game	Difference	EC game
Germany	13,708,647	−8	13,708,639
France	10,090,873	0	10,090,873
United Kingdom	10,039,263	0	10,039,263
Italy	9,743,561	0	9,743,561
Spain	7,362,225	+4	7,362,229
Poland	6,800,035	+4	6,800,039
Romania	4,871,079	+10	4,871,089
The Netherlands	4,048,155	+10	4,048,165
Greece	3,326,413	+10	3,326,423
Portugal	3,237,845	+10	3,237,855
Belgium	3,231,183	+10	3,231,193
Czech Republic	3,210,689	+10	3,210,699
Hungary	3,196,981	+10	3,196,991
Sweden	3,033,941	+10	3,033,951
Austria	2,911,521	+10	2,911,531
Bulgaria	2,877,541	+10	2,877,551
Denmark	2,530,351	+10	2,530,361
Slovak Republic	2,530,351	+10	2,530,361
Finland	2,509,871	+10	2,509,881
Ireland	2,332,655	+10	2,332,665
Lithuania	2,264,209	+10	2,264,219
Latvia	2,099,685	+10	2,099,695
Slovenia	2,051,511	+10	2,051,521
Estonia	1,962,711	+10	1,962,721
Cyprus	1,873,045	+10	1,873,055
Luxembourg	1,831,725	+10	1,831,735
Malta	1,824,817	+10	1,824,827

We should take into account that the number of non-empty coalitions in the scenario of 27 countries is 134,217,727 and the swings oscillate from 1,824,817 to 13,708,647, corresponding to Malta and Germany, respectively.

As a consequence of the data showed in Tables 2 and 3, the requirement of demanding at least four countries to block a decision complicates the procedure unnecessarily, because the change in the distribution of the power is insignificant.

The Banzhaf index of a nation is obtained dividing the number of decisive coalitions in which this nation participates by the total number of decisive coalitions. In this way, a distribution is obtained, between zero and one, which measures the capacity of decision to approve motions in an institution. If we multiply by a hundred this index, we obtain the percentage of the Banzhaf power.

In Table 4, the population and the Banzhaf power's percentages of the 25 nations of the European Union are given applying the Nice rule, the European Convention rule (at least 13 nations and at least 60% of the population) and the rule approved in the summit of Brussels to incorporate it to the European Constitution (at least 15 nations, with at least 65% of the population, and with more than 3 nations to block).

Fig. 1 shows a three-dimensional graph from the data included in Table 4 in order to display the mentioned losses and winnings of power of the 25 countries of the Union European with respect to the Nice, the European Convention and the European Constitution rules.

In Table 4 and Fig. 1, it can be observed the consequences of the rebellion carried out by the countries of smaller population in the summit of Brussels. Indeed, the 19 small and medium European countries win

Table 4
Population and Banzhaf power for the 25 EU members

Countries	Population	Nice rule	Convention	Constitution
Germany	18.158	8.5606	13.360	10.424
France	13.118	8.5600	9.4887	7.5805
United Kingdom	13.052	8.5600	9.4281	7.5395
Italy	12.610	8.5600	9.1807	7.3818
Spain	9.141	8.1221	7.0202	5.8233
Poland	8.408	8.1221	6.7677	5.5566
The Netherlands	3.562	4.2284	3.6395	3.7619
Greece	2.424	3.9103	2.9610	3.3285
Portugal	2.290	3.9103	2.9040	3.2907
Belgium	2.278	3.9103	2.9040	3.2907
Czech Republic	2.245	3.9103	2.8470	3.2528
Hungary	2.231	3.9103	2.8470	3.2528
Sweden	1.967	3.2725	2.7328	3.1773
Austria	1.775	3.2725	2.6188	3.1015
Denmark	1.184	2.3102	2.2730	2.8766
Slovak Republic	1.183	2.3102	2.2730	2.8766
Finland	1.145	2.3102	2.2155	2.8389
Ireland	0.872	2.3102	2.1002	2.7632
Lithuania	0.762	2.3102	2.0423	2.7255
Latvia	0.513	1.3292	1.8682	2.6139
Slovenia	0.439	1.3292	1.8102	2.5762
Estonia	0.298	1.3292	1.7523	2.5384
Cyprus	0.157	1.3292	1.6943	2.5010
Luxembourg	0.099	1.3292	1.6360	2.4637
Malta	0.087	0.9933	1.6360	2.4637

power and the 6 largest countries lose it, with respect to the European Convention rule. Moreover, the smallest countries (Malta, Luxembourg, Cyprus, Estonia, Slovenia and Latvia) win more and the largest (Germany, France, the United Kingdom, Italy, Spain and Poland) lose more. Finally, the power of Latvia, Slovenia, Estonia, Cyprus, Luxembourg and Malta increases with regard to the Nice and Convention rules; being Spain and Poland the only countries that lose if we compare the Nice rule with the European Convention rule and this rule with the Constitution rule.

5. The power of the European citizens

The Banzhaf power index measures the power of each nation in the Council of Ministers of the European Union, when it makes decisions using weighted voting rules. However, measuring the decision power of each European citizen is more difficult. The reason is that the participation of the citizens in the decision processes of the European Union is a procedure that has two phases. In the first one, we vote in favour of representatives that take a collective decision to the European institutions, being these institutions those that decide in a second phase, assigning a number of votes to each national representative and a quota which must be reached to approve a motion.

The distribution of votes proportional to the population of a country can seem the best method so that each citizen's vote will be egalitarian. However, this reasoning is erroneous because the individuals vote through a representative, forced by the vote of a majority group of citizens of his/her country. For example, let us suppose that the country A has 50 million voters and the country B has 49 million and we assign to the delegates of each country a vote for each million of voters. If we use the simple majority rule, 25 million

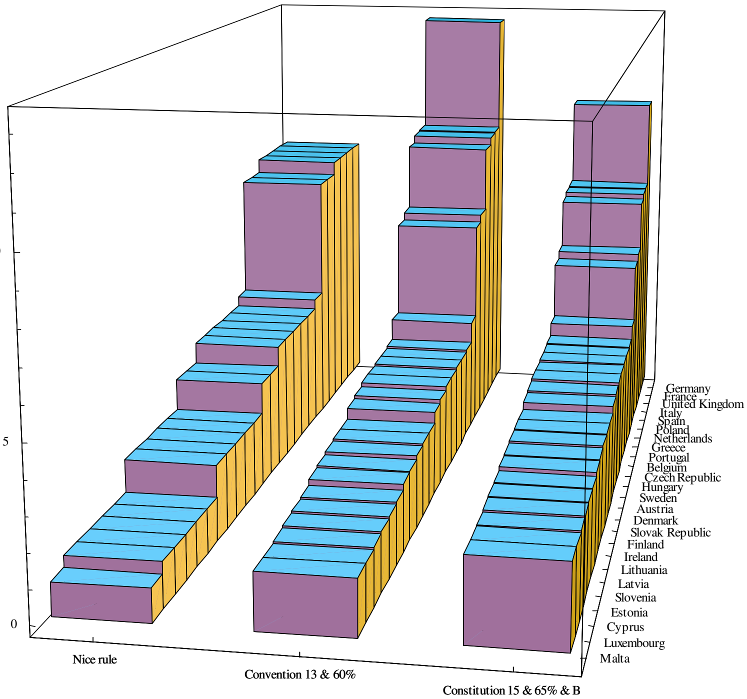

Fig. 1. Three-dimensional graph from the data included in Table 4.

plus one of voters of A are enough so that the delegate of A will use his/her 50 votes to approve a directive that forces a total community of 99 million. Then, this voting procedure gives more power to the voters of A and it permits imposing the decision of a minority.

Lionel Penrose [15] proposed the following mathematical model to analyze voting systems of this type. Let us consider several disjoint assemblies of voters and let N be the union of all the assemblies. Each assembly has a voting system in which each voter has a vote and the fixed quota is the absolute majority. Moreover, suppose that each delegate of an assembly votes in favour of a proposal if most of voters of this assembly have supported it and the delegate votes against otherwise. The following mathematical reasoning, that uses the Stirling approach formula, allows deducing that the *probabilistic Banzhaf index* of a voter in his/her assembly is inversely proportional to the square root of the number of voters of his/her assembly, whenever this number is large enough.

The standard method to analyze the power of a single voter is the following. We consider a weighted majority game (N, v) in which all of the n weights are 1 and the quota is $q = \lceil \frac{n+1}{2} \rceil$. In this game a voter i is a swing in a winning coalition S if and only if $i \in S$ and $|S| = q$. Then the number $b_i(v)$ of times that i is a swing is equal to

$$\binom{n-1}{q-1} = \frac{(n-1)!}{(q-1)!(n-q)!} = \begin{cases} \frac{(2k-1)!}{k!(k-1)!} & \text{if } n = 2k, \\ \frac{(2k)!}{(k!)^2} & \text{if } n = 2k+1. \end{cases}$$

Using Stirling's approximation formula

$$n! \sim \sqrt{2\pi n}\mathrm{e}^{-n}n^n,$$

we obtain

$$\frac{(n-1)!}{(q-1)!(n-q)!} \sim \begin{cases} \sqrt{\frac{2}{\pi n}}2^{n-1} & \text{if } n = 2k, \\ \sqrt{\frac{2}{\pi(n-1)}}2^{n-1} & \text{if } n = 2k+1. \end{cases}$$

Thus, the swing probabilities of voter i is the number

$$\beta'_i(v) = \frac{\binom{n-1}{q-1}}{2^{n-1}} \sim \sqrt{\frac{2}{\pi n}}$$

if n is sufficiently large.

Definition 3. The probabilistic Banzhaf index is the vector

$$\beta'(v) = \frac{1}{2^{n-1}}(b_1(v), \ldots, b_n(v)).$$

This vector is named the Banzhaf measure of voting power by Felsenthal and Machover [8, p. 39]. These authors showed that

$$\beta'_x \sim \beta'_i \sqrt{\frac{2}{\pi n_i}}$$

and obtained the following result for the model of two-tier voting system [8, p. 66].

Theorem 7. *The probabilistic Banzhaf indices β'_x are equal for all voters if and only if the probabilistic Banzhaf indices β'_i of the delegates are proportional to the respective $\sqrt{n_i}$.*

In the model proposed by Felsenthal and Machover [8], if the probabilistic Banzhaf index of the voting system is proportional to the square root of the population of its country, then the power of a European citizen is egalitarian. By using the properties of the probabilistic Banzhaf index for a compound game, Laruelle and Widgrén [13] proposed a method to obtain voting weights which lead to a fair allocation of power in the European Union.

The square root rule follows from the following *composition* principle: (the power of a voter i in a compound game) $\equiv$ (i's power in the first-tier game) $\times$ (the power of i's delegate in the second-tier game). Dubey et al. [7] proved that there exists one, and only one, power index satisfying the positivity, transfer and composition axioms, and it is the probabilistic Banzhaf index. The most crucial axiom of this characterization is composition. Therefore, our preference for the model of *I-power* justify the square root rule to analyze the power of the European citizens. Gelman et al. [12] presented data from the Electoral College of the United States and several European national elections which imply that the probability of a decisive vote is proportional to $n_i^{-0.9}$ which is much closer to $1/n_i$ than to $1/\sqrt{n_i}$. The reason why the square root rule does not hold is that the mentioned elections are consequence of local, regional and state swings. However, the decision rule adopted in the European Constitution is a two-tier voting system in which the two component games are independent and there is not influence of regional swings.

Next, Figs. 2 and 3 show two-dimensional data. The first coordinate, with values on the horizontal axis, is the square root of the population and the second coordinate, with values on the vertical axis, corresponds to the probabilistic Banzhaf index with the Nice rule. The figures correspond to the scenarios of 25 and 27 countries, respectively. With these data, the lineal functions of regression, whose graphs are straight lines, have been calculated. The points of the straight line are those in which the power is egalitarian. As the points of each straight line represent an equal distribution of the power, we conclude that the citizens from

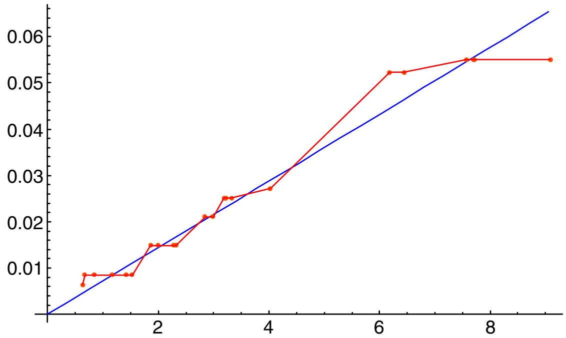

Fig. 2. The Nice rule for the 25-EU.

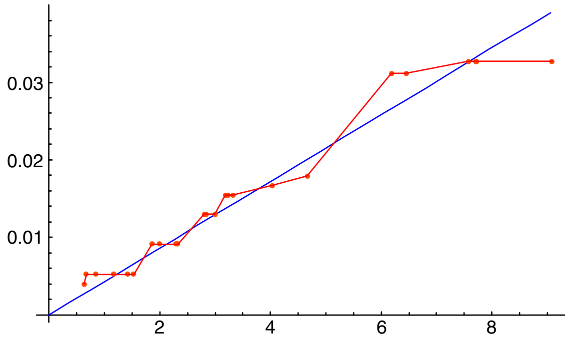

Fig. 3. The Nice rule for the 27-EU.

Germany (the point located more to the right) have less power because it is located under the straight line; France, the United Kingdom and Italy are on the straight line; while Spain and Poland are above it. The rest of the countries oscillate around the straight line corresponding to the equality of power.

In Figs. 4 and 5, the two-dimensional data and their lineal functions of regression for the scenarios of 25 and 27 countries are showed, these data are calculated with the rule of the double majority (at least 15 nations with at least 65% of the population and with 4 or more nations to block) approved in the summit of Brussels by the European Council. In the scenario of 25 countries we can notice the increase of power of

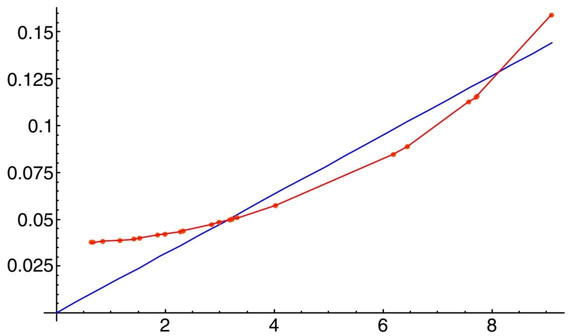

Fig. 4. The Constitution rule for the 25-EU.

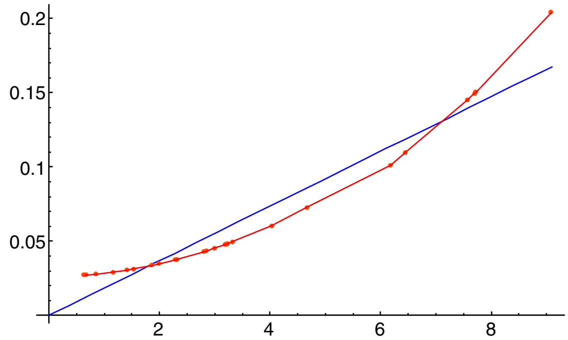

Fig. 5. The Constitution rule for the 27-EU.

the Germans, the losses of the French, British, Italians and Dutch; being the Spanish and Poles who have the biggest decrease of power. Analyzing the scenario of 27 countries, we deduce that the Constitution rule increases even more the power of the Germans, gives a superior power to the egalitarian one to the French, British, Italians and to the citizens of the six countries with less population. This rule also provides the inhabitants of the 17 remaining countries an inferior power to the egalitarian distribution.

Slomczynski and Zyczkowski [17] have worked out the voting rule called *Penrose-62* that consists of giving to each nation a proportional vote to the square root of its population and establishing a quota equal to 62% of the population. Their calculations proof that, with the rule *P-62*, the probabilistic Banzhaf index of each European nation is almost proportional to the square root of its population. In view of these results, a group of researchers in voting theory have written a letter to the governments of the member states of the European Union so that the basic democratic principle, that any citizen of the European Union has the same power of decision, will be respected. As the rule approved in Brussels and incorporated to the Constitution European disobeys this principle, we have proposed that the rule *Penrose-62* is adopted. With this voting system we will be able to reach the equality in the decision power of all the European citizens.

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