Preface

The aim of this book is to analyze conflict situations in which two or more players can make coalitions and obtain prizes and penalties. This approach to situations of competition and cooperation was given in the seminal treatise by John von Neumann and Oskar Morgenstern Theory of Games and Economic Behavior [242, 1944]. Cooperative game theory has succeeded in providing a lot of applications of game theory. I quote from the Robert Aumann’s interview realized by Eric van Damme [52]:

“Cooperative theory is actually doing quite well. I’ve already said in this interview that many of the most interesting applications of game theory come from the cooperative side::: Mike Maschler discussed over 30 sign...cant contributions to the cooperative theory that have been produced over the last few years::: On the cooperative side, there are three or four central solution concepts — value, core, nucleolus, stable sets— but much less flexibility in constructing the model. The model is much better defined.”

The classical hypothesis in cooperative game theory is that all coalitions of players are possible. However, this assumption is not realistic because cultural, ideological and structural issues will prevent the formation of some coalitions. Models of cooperative games in which cooperation among players depends on their communication were proposed by Myerson [182], Owen [195], and Borm, Owen and Tijs [26]. These models provide a worth for the feasible coalitions \( S \subseteq F \) and define a restricted game on all coalitions \( S \subseteq 2^N \) by additive extension with the maximal components of \( S \). The paper Cooperation and communication restrictions: a survey [27] by Borm, van den Nouweland and Tijs, is a guide for the above model in the context of imperfect or incomplete information and communication in economic organizations. Myerson [183] introduced hypergraphs in communication situations and van den Nouweland, Borm and Tijs [188] defined the interaction sets and obtained the axiomatic characterizations of the position and Myerson values.

A more general model, the partitioning games, was developed by Kaneko and Wooders [138]. They present the superadditive extension of a game on feasible coalitions, by using maximal components. There is another extension by Faigle and Kern [78], the cooperative games under precedence constraints. In this model the games are defined on distributive lattices of subsets of players. The core of a game \( v : F \rightarrow R \) was investigated by Faigle [77], who obtained necessary and sufficient conditions for the existence of the core from a general sandwich theorem for set functions.
Shapley [212, 1953] established an allocation rule satisfying a few simple axioms (see a survey in Roth [208]) and introduced [217] the class of convex games. For these games, the optimal allocation in the core can be obtained by the greedy algorithm, when the criterion is linear (see Fujishige [96]). With respect to the optimization methods, Grötschel, Lovász and Schrijver [116] observed that:

"Historically, there is a close connection between geometry and optimization. This is illustrated by methods like the gradient method and the simplex method, which are associated with clear geometric pictures. In combinatorial optimization, however, many of the strongest and most frequently used algorithms are based on the discrete structure of the problems: the greedy algorithm, shortest path and alternating path methods, branch-and-bound, etc. In the last several years geometric methods, in particular polyhedral combinatorics, have played a more and more profound role in combinatorial optimization as well."

In this volume, we will analyze games defined on combinatorial structures, i.e., a set system over a set of players. In many situations we will work in a closure space. Let \( N = \{1, \ldots, n\} \) be a set of players. By a closure operator \( \mu : 2^N \rightarrow 2^N \) we mean an operator satisfying the standard closure axioms:

\[
A \subseteq A; \quad A \subseteq A; \quad A \subseteq A \subseteq B \implies A \subseteq B.
\]

A set \( S \subseteq 2^N \) is said to be closed if \( S = S \). The pair \((N; \mu)\) is a closure space [113]. Examples of closure operators are the spanning operator of linear algebra and all convex hull operators. Matroids were introduced in 1935 by Whitney who conceived a matroid as a generalization of a matrix. The closure operator of a matroid and the spanning operator of linear algebra satisfies the Steinitz-MacLane exchange property:

\[
x; y \in A \quad \text{and} \quad y \notin A \implies x \in A \quad \text{for every} \quad A \subseteq N:\n\]

Convex geometries are closure spaces with the anti-exchange property:

\[
x; y \in A \quad \text{and} \quad y \notin A \implies x \notin A \quad \text{for every} \quad A \subseteq N:\n\]

The last property is a combinatorial abstraction of the convex closure in Euclidean spaces. We call the closed sets in a convex geometry convex sets. A common framework for matroids and convex geometries is found by the greedoids, structures introduced by Korte, Lovász and Schrader [144].

In a cost allocation problem there is a finite set of users who cooperate in a joint project. The problem is how to allocate the cost among the players in terms of a rule with properties such as efficiency, anonymity and stability (see Curiel [51]). Vectors in the core of the cost game are the best cost allocations.
when the core is nonempty. The min-cost spanning tree game is a balanced game and a core vector for this game is constructed from a minimal spanning tree by Granot and Huberman [109]. However, this does not mean that we shall be able to characterize all the extreme points of the core. Kuipers [148] showed that in the subclass of information graph-games, the extreme points are the marginal allocations of the associated submodular game. Nagamochi, Zeng, Kabutoya and Ibaraki [185] proposed a new minimum forest game on an edge-weighted graph and extended this game to a weighted matroid.

Combinatorial cooperative games are games such that the value $v(S)$ of the coalition $S \subseteq N$ is the optimal value of a combinatorial optimization problem defined by $S$ (see Faigle and Kern [79]). Matching games, simple flow games, traveling salesman games, sequencing games and delivery games (associated with the Chinese postman problem) have been considered.

Chapters 1 through 4 constitute a review of mathematical concepts from the Cooperative Game Theory, Graph Theory, Linear and Integer Programming, Combinatorial Optimization, Discrete Convex Analysis and Computational Complexity. The table of contents is a short guide to the topics and methods treated in this book. In Chapters 11 and 12, several notebooks are presented with the system Mathematica by Wolfram [248] in the contexts of the packages DiscreteMath (Skiena [221]) and Cooperative (Carter [34]). There will also be found in the book several research projects. These are intended to offer new ideas that the reader should consider with caution.

We hope that this book will be of interest to graduate students with some experience in game theory or mathematical programming and professional researchers in game theory, operational research and its applications in Economic Theory, and the Political and Social Sciences. We would also like this volume to be especially useful for professionals who are interested in models for understanding and managing conflicts: management and operational research scientists, political and military scientists as well as professional negotiators.

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Jesús Mario Bilbao Arrese
Seville, Spain, December 1999
http://www.esi2.us.es/~mbilbao/
E-mail: mbilbao@cica.es