4 Network Formation and Coordination: Bargaining the Division of Link Costs

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This paper presents a model of network formation in which links are costly. We endogeneize the part of the cost supported by each of the players involved in a bilateral link. In this sense we consider that these sharings result from bargaining. We study this process in a context of coordination games. We show that, if this cost is not too high, players coordinate either in the risk-dominant action or in the efficient one: if costs of forming links are higher than the risk-dominance premium the efficient action is selected; meanwhile, if they are lower, the risk-dominant action prevails.

There are social and economic situations in which the existence of some kind of connections between the agents is necessary to interact. We can think for example on information transmission: agents need some way of communication in order to be able to exchange information. In many cases the establishment and maintenance of these connections is costly. We model a situation in which the benefits from interacting is related to coordination, that is, any two players who establish a link benefit if they are coordinated in the same action. The main feature of our model is how the agents who form a link share the cost it involves. We propose that this division results from bargaining; in this sense, we make the agents' shares of the link cost endogenous. The model deals with the choice of a standard in a population (e.g. PC vs. Mackintosh) and with the network formation, that is, given the choices on standards, each agent decides who she wants to interact with (i.e. to form links). The earnings of the interaction between two agents (i.e. of forming a link) are represented by the payoffs of a $2 \times 2$ symmetric coordination game in which we identify the actions with the standards chosen. This game is characterized by two pure strategy Nash equilibria, one efficient and the other risk-dominant. The formation of a link is costly and we consider that the part of the cost each of the involved agent supports results from bargaining. Thus, in this model each player will first decide a standard and then, each possible pair of players enters in a bargaining process in which they have to agree on how to share the cost (and form a link), or reach the outside option of not forming the link.

We propose the Nash Bargaining solution to distribute the cost. We find that this game presents multiplicity of equilibria. Therefore we introduce a dynamics in which, from any initial state, each period players receive revision opportunities. We assume a best-response adjustment to update strategies.
We analyze the set of limit states of this process and we get that the initial multiplicity persists. To deal with the equilibrium selection we use stochastic stability techniques. We find a threshold for the link cost that coincides with the risk-dominance premium. If the cost of the link is lower than this threshold we get that all the population coordinates in the (inefficient) risk-dominant standard. If the cost of the link is higher, efficiency is achieved, provided that the cost is not so high that no link can be profitable for both players involved.

The study of networks has been increasingly considered in the literature in the last years. Specially relevant in this field is the work by Jackson and Wolinsky (1996), who study the stability and efficiency of social and economic networks; in their work they do not formally model the procedure through which a graph is formed. There are also studies that explicitly analyze the dynamic process of network formation. Consider for example Bala and Goyal (2000), Jackson and Watts (1998), and Watts (2001). In these models agents only decide about link formation and there are no other actions that influence their payoffs. Bala and Goyal (2000) develop a noncooperative model of network formation considering both directed and non-directed networks. They show convergence to strict Nash networks.

Specifically, if we consider models in which the formation of links is costly, in the literature we find two ways to tackle with the link cost: (i) the one-sided links models, that are characterized by the fact that the agent who proposes to form a link will completely cover the cost; (ii) the two-sided links models, in which each of the two agents involved in the link will share the cost in equal amount. In the setup of social coordination games (where our research fits) Goyal and Vega-Redondo (2000) is framed in the first kind of models; among the second ones we find Jackson and Watts (1999) and Droste, Gilles and Johnson (2000), in which a spatial location of agents is introduced. Both kind of models (one-sided and two sided) seem to be questionable since it is reasonable to argue that when two players have the possibility to form (or maintain) a link, the one who will get a greater payoff from it will be willing to cover a higher part of the cost it involves. We propose the Nash solution to distribute the cost of a link in this setup of bilateral coordination games. This endogeneity of the distribution of the cost provides two important advantages to our model over the former ones: (i) now, whenever a link is profitable (the sum of the link-payoffs of the two agents is higher than the link-cost) the link will be formed, which in fact will result in a higher connectivity; and (ii) the cost supported by each player in a certain link will depend on the relative payoff of both agents involved in it. We get that our results are related to the first kind of models.